1-2018

Closed-loop supply chain network design with multiple transportation modes under stochastic demand and uncertain carbon tax

Ali Haddadsisakht
Wise Systems, Inc.

Sarah M. Ryan
Iowa State University, smryan@iastate.edu

Closed-loop supply chain network design with multiple transportation modes under stochastic demand and uncertain carbon tax

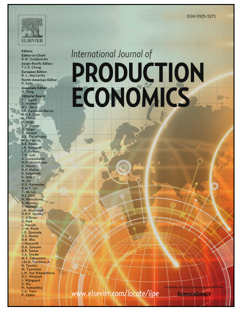
Abstract
We optimize the design of a closed-loop supply chain network that encompasses flows in both forward and reverse directions and is subject to uncertainty in demands for both new and returned products. The model also accommodates a carbon tax with tax rate uncertainty. The proposed model is a three-stage hybrid robust/stochastic program that combines probabilistic scenarios for the demands and return quantities with uncertainty sets for the carbon tax rates. The first stage decisions are facility investments, the second stage concerns the plan for distributing new and collecting returned products after realization of demands and returns, and the numbers of transportation units of various modes are the third stage decisions. The second- and third-stage decisions may adjust to the realization of the carbon tax rate. For computational tractability, we restrict them to be affine functions of the carbon tax rate. Benders cuts are generated using recent duality developments for robust linear programs. Computational results show that adjusting product flows to the tax rate provides negligible benefit, but the ability to adjust transportation mode capacities can substitute for building additional facilities as a way to respond to carbon tax uncertainty.

Keywords
Affinely adjustable robust counterpart, Closed-loop supply chain, Scenario-based optimization, Benders decomposition, Semi-definite programming

Disciplines
Industrial Engineering | Operational Research | Systems Engineering

Comments
This is a manuscript of an article published as Haddad-Sisakht, Ali, and Sarah M. Ryan. "Closed-loop supply chain network design with multiple transportation modes under stochastic demand and uncertain carbon tax." *International Journal of Production Economics* (2018). 10.1016/j.ijpe.2017.09.009. Posted with permission.

Accepted Manuscript



Closed-loop supply chain network design with multiple transportation modes under stochastic demand and uncertain carbon tax

Ali Haddad-Sisakht, Sarah M. Ryan

PII: S0925-5273(17)30293-1

DOI: 10.1016/j.ijpe.2017.09.009

Reference: PROECO 6822

To appear in: *International Journal of Production Economics*

Received Date: 13 September 2016

Revised Date: 8 June 2017

Accepted Date: 12 September 2017

Closed-loop supply chain network design with multiple transportation modes under stochastic demand and uncertain carbon tax

Ali Haddad-Sisakht[a], Sarah M. Ryan[b,*]

[a]*Operations Research Scientist, Wise Systems, Inc. Cambridge, MA*
[b]*Joseph Walkup Professor, Department of Industrial and Manufacturing Systems Engineering, Iowa State University, Ames, IA*

Abstract

We optimize the design of a closed-loop supply chain network that encompasses flows in both forward and reverse directions and is subject to uncertainty in demands for both new and returned products. The model also accommodates a carbon tax with tax rate uncertainty. The proposed model is a three-stage hybrid robust/stochastic program that combines probabilistic scenarios for the demands and return quantities with uncertainty sets for the carbon tax rates. The first stage decisions are facility investments, the second stage concerns the plan for distributing new and collecting returned products after realization of demands and returns, and the numbers of transportation units of various modes are the third stage decisions. The second- and third-stage decisions may adjust to the realization of the carbon tax rate. For computational tractability, we restrict them to be affine functions of the carbon tax rate. Benders cuts are generated using recent duality developments for robust linear programs. Computational results show that adjusting product flows to the tax rate provides negligible benefit, but the ability to adjust transportation mode capacities can substitute for building additional facilities as a way to respond to carbon tax uncertainty.

Keywords: Affinely Adjustable Robust Counterpart, Closed-Loop Supply

*Corresponding author, Tel: +1 515 294 4347, Email: smryan@iastate.edu

Chain, Scenario-Based Optimization, Benders Decomposition, Semi-definite Programming.

1. Introduction

To reduce the negative environmental impacts from supply chains, legislation and social concerns have been motivating firms to plan their supply chain structures for handling both forward and reverse product flows. In a closed-loop supply chain (CLSC) network, forward flows satisfy demands for new products, while reverse flows represent collection and remanufacturing or recycling of returned products. Product returns may occur due to retailer overstocks and consumer dissatisfaction, extended producer responsibility legislation, or the potential profits derived from remanufacturing and resale. Companies that coordinate their reverse flows with forward flows are usually more successful with their return supply chains (Guide & Van Wassenhowe, 2002). Network design is one of the most important strategic decisions in a firm's CLSC management. As the CLSC network is expected to be in use for a considerable amount of time, the firm should consider all the possible factors that will affect the design decisions. Designing such a network involves long-term decisions to invest in fixed facilities as well as more flexible decisions, such as transportation capacities and product flows. Transportation choices include various modes available, either by purchasing or leasing vehicle fleets or by contracting with external providers.

One source of the environmental impacts is the carbon emissions from transporting products. Returning products for recycling or remanufacturing increases reuse. However, imposing a cost on carbon emissions can reduce the return flow as the return transportation cost is effectively increased (Allevi et al., 2016; Xu et al., 2017). Much research has been proposed to mitigate the adverse environmental effects of freight transportation, particularly CO_2 emissions (Hickman & Banister, 2011). One approach involves decisions concerning the choice among modes with varying emission rates, capacities, and costs (Mallidis et al., 2012). According to a survey, 26% of CO_2 emissions were generated by transportation activities in 2014 (U.S. Environmental Protection Agency, 2016b). International trade liberalization contributes to significantly more transportation of products in global supply chains (Mallidis et al., 2012). These trades employ different modes of transportation such as road, rail, air, and water, each of which has a different rate of greenhouse gas (GHG) emissions. Among them, freight transportation modes account for nearly 57% of CO_2 emissions. Light trucks were responsible for 17% of CO_2 emissions while medium- and heavy-duty trucks contributed 23% on average between 2010 to 2014 (U.S. Environmental Protection Agency, 2016a).

With concern over global climate change, regulations on carbon emissions resulting from industries such as transportation and power generation have been developed by policy-makers in different nations. For example, in 2005 the European Union instituted a carbon emission trading scheme (EU ETS) for the energy-intensive industries with the aim of reducing GHG emissions by at least 20% below 1990 levels (Behringer et al., 2009). In addition, China, which is one of the world's largest emitters of GHG, has announced in recent years that the Ministry of Finance may levy taxes on CO_2 emissions (Xinhuanet, 2013). As of January 2011, the US Environmental Protection Agency (EPA) has power to regulate the carbon emissions of companies operating in the US. In the past, the federal government has tended to emphasize "command and control" regulatory approaches to control pollutants. For the US to reduce its GHG emissions, most environmental policy analysts agree it must use market-based environmental mechanisms. The two main market-based options are a carbon tax and a cap-and-trade system of tradable permits for emissions (Metcalf, 2009), with the tax proposals currently receiving more attention.

Motivated by the effect of carbon emission regulations on a firm's CLSC network design (Fahimnia et al., 2013), this paper investigates the effect of an uncertain carbon tax rate on the network design decisions. In major carbon-emitting nations such as the US, there is uncertainty associated with the carbon tax rates once implemented. The tax rates elsewhere vary considerably. For example, tax rate of Finland was \$30/metric ton CO_2 in 2008 while British Columbia, starting from \$9.50/metric ton in 2008, increased to \$30 in 2012 (Sumner et al., 2009). Some US federal agencies including the EPA estimated the social cost of carbon to be \$36 in 2015 (U.S. Environmental Protection Agency, 2017). Therefore, how uncertainty concerning emission tax rates should affect the network configuration, choice of transportation modes and planned magnitudes of product flows while minimizing the overall cost is worthy of investigation.

In addition to carbon tax uncertainty, we also consider the uncertainty associated with demand and return quantities. The modeling contribution of this paper is the formulation of a three-stage hybrid robust/stochastic program (Keyvanshokooh et al., 2016) with multiple scenarios for the demands and return quantities and an uncertainty set for the carbon tax rate. The first stage includes binary decisions of investing in candidate facilities as a long-term strategy that is robust to carbon tax regulation and optimizes the expected cost of satisfying demands and collecting returns. Planned product flows are the second stage decisions that optimally balance the tradeoffs between transportation cost and emission-related operational costs. Transportation capacities of various modes are the third stage decisions that, along with the product flows, can adjust to the carbon tax rate once it is revealed. While several sources of uncertainty have been studied previously in CLSC network design, most of the literature assumes high levels of knowledge about their probability distributions. We focus on the epistemic uncertainty associated with new products in regions where carbon taxes have not been levied before. Therefore we consider the scenarios to broadly represent product acceptance and likelihood of product return rather than high-frequency variability. The planned product flows are tactical decisions that distribute the demands and returns among facilities and balance the tradeoffs between transportation costs and penalties for not collecting all returns. Because implementation of a carbon tax could be delayed, we assume the decisions of how to transport new and returned products are delayed until after the tax rate is known. While our initial model also allows product flows to adjust to the tax rate, numerical studies indicate that the benefit of doing so does not justify the additional computational effort. Thus, we focus on the formulation in which only transportation capacities of different modes are adjustable to the carbon tax.

Including a large number of scenarios for demands and returns in large-scale instances renders the solution procedure computationally cumbersome. Therefore, we apply a multi-cut version of Benders decomposition (BD) to solve the hybrid robust/stochastic model by decomposing the problem into master and sub-problems. The methodological contribution of this paper is to formulate the Benders cuts using the dual solutions of robust counterpart (RC) and affinely adjustable robust counterpart (AARC) sub-problems, which we obtain using recent duality results.

The results of numerical case studies show how the optimal number and locations of opened facilities respond to uncertainty in the demand and return quantities. In addition, we observe how the choice of transportation modes responds to different carbon tax uncertainty levels and the extent to which adjustability of transportation capacities to carbon tax rates is beneficial. The AARC solution exhibits higher utilization of the transportation modes with higher capacity and lower emission rates than the non-adjustable RC solution. Also, in some cases, allowing transportation capacities to respond to the carbon tax rate reduces the investment in fixed facilities.

A brief review of the recent literature follows in Section 2. In Section 3, we introduce our CLSC network design formulations. We present computational results in Section 4 and finally conclusions as well as future research directions in Section 5.

2. Literature Review

Carbon emission regulations on transportation have been considered in deterministic supply chain models. For example, Benjaafar et al. (2013) presented and modified traditional supply chain models to include carbon footprint along with other costs. They examined different regulatory emissions such as cap-and-trade and carbon tax and presented the effect of their parameters on costs and emissions. Pan et al. (2010) explored the environmental impact of pooling of supply chain resources at a strategic level and extracted the emission functions of two transport modes, rail and road, using a French case study. Hoen et al. (2010) investigated the effect of cap-and-trade and company-wide (hard constraint on emissions) regulation on transportation mode decisions. Furthermore, they analyzed the effect of considering emission costs or emission in their model, and they found that emission cost penalties have only a small effect on transport mode selection compared to constraints. However, they did not consider the effect of emission cost parameters on transportation mode decisions. More research includes the investigation of Bloemhof-Ruwaard et al. (2011) on the environmental impact of inland navigation (transportation by canals or rivers) compared to inland transport modes, which identified that the road transport mode is the biggest contributor of hazardous gas emission. Fu & Kelly (2012) evaluated the impacts of different transportation tax policies for carbon emission in Ireland. Their results suggested that the fuel based carbon tax is better than either a vehicle registration tax or motor tax in terms of tax revenue, carbon emission reductions, and social welfare, but worse than the latter in terms of household utility and production costs. Zakeri et al. (2015) presented an analytical supply chain planning model to examine the supply chain performance under carbon taxes and carbon emissions trading. They found that the carbon tax is more worthwhile from an uncertainty perspective as emissions trading costs depend on numerous uncertain market conditions. These studies have not considered the effect of carbon tax uncertainty on the choice among transportation modes.

CLSC design problems have been relatively well-studied (Zeballos et al., 2012; Vahdani et al., 2012; Vahdani & Mohammadi, 2015), but carbon emissions have been considered only recently, and mostly in deterministic models. Paksoy & Ozceylan (2011) proposed a general CLSC network configuration that handles various costs including emission costs for transportation activities in a completely deterministic environment for all parameters. Chaabane et al. (2012) proposed a generic mathematical model to design and plan a CLSC based on the life cycle assessment methodology. Their model considers an emission-trading scheme that caps GHG emissions and impose mandatory targets for recycling products at the end of their life in the aluminum industry. Fahimnia et al. (2013) analyzed different costs and environmental influences on the tactical-operational planning contingencies that included carbon emissions in terms of dollars for the first time. They developed and tested a mixed integer-linear programming (MILP) formulation of an actual case in Australia. Fareeduddin et al. (2015) proposed a CLSC design problem that considered location, production technology and transportation mode selection related decisions to investigate the impact of carbon regulatory policies such as carbon cap, carbon tax, and carbon cap-and-trade on supply chain operations. They found that carbon tax policy provides more flexibility but imposes a high financial burden to reach a given emissions target compared to the other two policies. The work of Tao et al. (2015) is related to CLSC network equilibrium comprising manufacturers, retailers, demand markets and recyclers comparing periodic and global mandatory carbon emission constraints during manufacturing/remanufacturing. Allevi et al. (2016) formulate and optimize the equilibrium state of a CLSC network problem assuming that manufacturers are subject to the EU-ETS and a carbon tax is imposed on truck transport. They analyzed how carbon policies and regulations affect product flows, carbon emission generation, and recycling processes in CLSC. Xu et al. (2017) analyzed the effect of carbon emissions on the design of both hybrid and dedicated CLSCs where in the hybrid version, the facilities for forward logistics can be used for reverse logistics also. They compared both economical and environmental impacts of carbon emission policies such as carbon cap, carbon tax, and carbon cap-and-trade. They found that the hybrid CLSC is more emissions-efficient when the carbon tax is introduced.

Uncertainty in carbon emission regulations has been investigated in CLSC network design only by Gao & Ryan (2014), who considered a robust formulation of a multi-period capacitated CLSC network design problem while considering two regulations for carbon emissions. They integrated stochastic programming and robust optimization to deal with uncertainty in demands and returns as well as parameters of regulations on carbon emissions from transportation by different modes. They observed that, as the uncertainty level in the carbon tax increases, more facilities are opened and more capacity of modes with lower emission rates is used. Their model did not allow for the allocation of capacity among transportation modes to adjust to the carbon tax rate. Our model incorporates this adjustability to obtain a less conservative design. We show that by allowing adjustability unlike in the Gao & Ryan (2014) model, the same number of facilities can accommodate more uncertainty.

To model an uncertain carbon tax rate, we formulate the RC of the optimization problem with uncertain parameters whose distribution functions are unknown or difficult to determine. This approach was first proposed by Soyster (1972) and further developed by Ben-Tal & Nemirovski (1998, 1999, 2000) as well as independently by El Ghaoui & Lebret (1997); El Ghaoui et al. (1998). The more recent papers proposed tractable solution approaches to special cases of robust counterparts in the form of conic quadratic problems with less conservative solutions than the Soyster (1972) approach. Ben-Tal et al. (2004) defined the adjustable robust counterpart (ARC) and more tractable AARC models with adjustable variables that tune themselves to the values of uncertain parameters described by certain forms of uncertainty sets. They defined conditions under which the solutions of RC and ARC are equal. Haddad-Sisakht & Ryan (2016) established conditions under which affine adjustability may lower the optimal cost of the RC solution. In our three-stage model, we integrate a scenario-based optimization for product uncertainties with an AARC for tax rate uncertainty. To our knowledge, the generation of Benders cuts from the duals of the RC and AARC formulations has not been done previously.

3. CLSC Design Model

First we present a deterministic model for optimizing facility investments, transportation quantities and capacities of different transportation modes. We assume a carbon tax rather than a cap-and-trade system, since this is politically

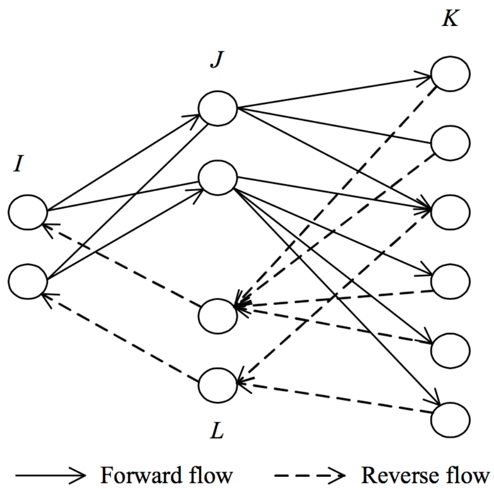


Figure 1: Closed-loop supply chain network structure

more likely in the US. Moreover, it may be the only feasible way to regulate emissions from transportation because of the large number of entities involved. The closed-loop supply chain network is denoted by $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ is the set of arcs. The node set $\mathcal{N} = \mathcal{P} \cup \mathcal{K}$, where $\mathcal{P}$ is a set of potential facilities consisting of factories $\mathcal{I}$, new product warehouses $\mathcal{J}$, collection centers for returned products $\mathcal{L}$; i.e., $\mathcal{P} = \mathcal{I} \cup \mathcal{J} \cup \mathcal{L}$; and $\mathcal{K}$ is the set of retailers. Let $\mathcal{M}$ be the set of transportation modes available for the supply chain. The arc set $\mathcal{A} = \{ij : (i \in \mathcal{I}, j \in \mathcal{J}), (i \in \mathcal{J}, j \in \mathcal{K}), (i \in \mathcal{K}, j \in \mathcal{L}), (i \in \mathcal{L}, j \in \mathcal{I})\}$ (see Figure 1 for the network topology). The closed-loop supply chain configuration decisions consist of determining which of the processing facilities to open. Let binary variable y_i be the decision to open the processing facility $i \in \mathcal{P}$ and x_{ij}^m be the number of units of product transported from node i to node j using transportation mode m, where $ij \in \mathcal{A}$ and $m \in \mathcal{M}$. Decision variables t_{ij}^m denote the number of units of transportation mode $m \in \mathcal{M}$ for which to contract on arc $ij \in \mathcal{A}$. Thus, t_{ij}^m is the amount of capacity, with associated fixed cost, made available to transport x_{ij}^m products.

In addition, the decision variables for unmet demands and discarded returns are denoted as z_k and e_k units of products respectively, for customer k. In this model, we do not consider keeping inventory in facilities across periods. We assume that manufacturers are responsible for processing returns after receiving them from collection centers, and we only consider a single product. The nominal deterministic mathematical model for CLSC network design can be stated as follows:

$$Z_{ND} = \min \sum_{i \in \mathcal{P}} c_i y_i + \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} h^m t_{ij}^m + \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} g^m \beta_{ij} x_{ij}^m + \sum_{k \in \mathcal{K}} (\theta z_k + \zeta e_k) + w\alpha \sum_{ij \in \mathcal{A}} \beta_{ij} \sum_{m \in \mathcal{M}} \tau^m x_{ij}^m \tag{1}$$

$$\text{s.t.} \quad \sum_{ij \in \mathcal{A}} \left(h^m t_{ij}^m + g^m \beta_{ij} x_{ij}^m + w\alpha \beta_{ij} \tau^m x_{ij}^m \right) \geq L^m, \qquad \forall m \in \mathcal{M} \tag{2}$$

$$\sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{M}} x_{jk}^m + z_k = d_k^n, \qquad \forall k \in \mathcal{K} \tag{3}$$

$$\sum_{i \in \mathcal{L}} \sum_{m \in \mathcal{M}} x_{ki}^m + e_k = d_k^o, \qquad \forall k \in \mathcal{K} \tag{4}$$

$$\sum_{i \in \mathcal{K}} \sum_{m \in \mathcal{M}} x_{ji}^m - \sum_{i \in \mathcal{I}} \sum_{m \in \mathcal{M}} x_{ij}^m = 0, \qquad \forall j \in \mathcal{J} \tag{5}$$

$$\sum_{i \in \mathcal{I}} \sum_{m \in \mathcal{M}} x_{ji}^m - \sum_{i \in \mathcal{K}} \sum_{m \in \mathcal{M}} x_{ij}^m = 0, \qquad \forall j \in \mathcal{L} \tag{6}$$

$$\sum_{j: ij \in \mathcal{A}} \sum_{m \in \mathcal{M}} x_{ij}^m - \eta_i y_i \leq 0, \qquad \forall i \in \mathcal{P} \tag{7}$$

$$w x_{ij}^m - W_m t_{ij}^m \leq 0, \qquad \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{8}$$

$$y \in \{0,1\}^{|\mathcal{P}|}, x \in \mathbb{R}_+^{|\mathcal{A}| \times |\mathcal{M}|}, t \in \mathbb{R}_+^{|\mathcal{A}| \times |\mathcal{M}|}, \quad z, e \in \mathbb{R}_+^{|\mathcal{K}|} \tag{9}$$

In this model, c_i denotes the investment cost (\$) for building facility $i \in \mathcal{P}$, h^m is the approximate fixed operating cost (\$/units of transportation) per unit of capacity of transportation mode m, g^m is the unit transportation cost (\$/units of product-km) of mode m, and β_{ij} is the distance (km) from node i to node j. The unmet demand cost is θ (\$/units of product) and the corresponding cost for discarded returns is ζ. In addition, α is the carbon tax rate (\$/ton) subject to an uncertain exogenous policy decision. In the last term of the objective function, w is the weight of product (tons/units of product), and τ^m is the carbon emission factor (tons/ton-km) for transportation mode m.

Constraints (2) introduce a lower bound L^m on the cost of mode m as determined by management. A constraint, such as (2), that guarantees at least minimal use of some transportation mode might reflect units of capacity already procured (Yuzhong & Guangming, 2012) or the desire to guarantee access to a mode that provides rapid delivery despite its higher emissions and cost (Turban et al., 2015). Contractual provisions might cause reluctance to change usage dramatically from previous periods. Or, usage above a threshold might gain a quantity discount. A lower bound on the cost of using a transportation mode is used, instead of a direct lower bound on t, because considering a minimal number of transportation units procured does not necessarily guarantee the use of that available mode for transportation. Considering a lower bound based on cost, as opposed to the number of transportation units, also could reflect how much a manager would like to spend on internal capacity rather than outsourcing. In addition, constraining cost as a continuous quantity is compatible with our neglect of integer restrictions on the units of transportation capacity to avoid computational complications. This constraint will be revisited in the adjustable RC in Section 3.2.

Constraints (3) and (4) compute met or unmet demands and collected returns, where d_k^n is the demand (units of product) for new products and d_k^o is the quantity of returns (units of product). Constraints (5) and (6) ensure that the warehouse and collection centers will not carry stocks across periods or incur backlogs. Constraint (8) requires that the product's weight does not exceed the total capacity of transportation mode m from node i to node j, where W_m denotes the weight limit (tons/units of transportation capacity) of mode m. Constraint (7) enforces capacity constraints of the processing nodes, where η_i denotes the capacity at node $i \in \mathcal{P}$. Finally, variable restrictions are given in

(9).

We incorporate uncertainty by elaborating a three-stage hybrid robust/stochastic program with multiple scenarios for the demands and returns as well as an uncertainty set for the carbon tax rate. The first stage variables determine long-term facility investments that are robust to both types of uncertainty. We describe the incorporation of probabilistic scenarios for demands and returns in Section 3.1. To incorporate carbon tax uncertainty, the robust counterpart of the recourse problem for each scenario is formulated in Section **??**, along with adjustable and affinely adjustable versions. Finally, the full three-stage formulations are described in Section **??**, along with dual formulations of the linear robust counterparts.

3.1. Stochastic program for CLSC design

In this subsection, we incorporate probabilistic scenarios for demands and return quantities. Letting $s \in \mathcal{S}$ denote a given realization with probability P_s, the nominal stochastic programming extension of (1)-(9) is as follows:

$$Z_{NS} = \min_{y \in \{0,1\}^{|\mathcal{P}|}} \sum_{i \in \mathcal{P}} c_i y_i + \sum_{s \in \mathcal{S}} P_s Q_N(y, s) \tag{10}$$

where the second stage of the stochastic program optimizes cost in a given scenario, assuming the nominal value, $\bar{\alpha}$, for the carbon tax rate:

$$\begin{aligned} Q_N(y, s) = \min_{x_s, t_s, z_s, e_s} & \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} h^m t^m_{ijs} + \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} g^m \beta_{ij} x^m_{ijs} \\ & + \sum_{k \in \mathcal{K}} (\theta z_{ks} + \zeta e_{ks}) + w\bar{\alpha} \sum_{ij \in \mathcal{A}} \beta_{ij} \sum_{m \in \mathcal{M}} \tau^m x^m_{ijs} \end{aligned} \tag{11}$$

$$\text{s.t.} \quad \sum_{ij\in\mathcal{A}} \left(h^m t_{ijs}^m + g^m \beta_{ij} x_{ijs}^m + w\bar{\alpha}\beta_{ij}\tau^m x_{ijs}^m\right) \geq L^m, \qquad \forall m \in \mathcal{M} \tag{12}$$

$$\sum_{j\in\mathcal{J}} \sum_{m\in\mathcal{M}} x_{jks}^m + z_{ks} = d_{ks}^n, \qquad \forall k \in \mathcal{K} \tag{13}$$

$$\sum_{i\in\mathcal{L}} \sum_{m\in\mathcal{M}} x_{kis}^m + e_{ks} = d_{ks}^o, \qquad \forall k \in \mathcal{K} \tag{14}$$

$$\sum_{i\in\mathcal{K}} \sum_{m\in\mathcal{M}} x_{jis}^m - \sum_{i\in\mathcal{I}} \sum_{m\in\mathcal{M}} x_{ijs}^m = 0, \qquad \forall j \in \mathcal{J} \tag{15}$$

$$\sum_{i\in\mathcal{I}} \sum_{m\in\mathcal{M}} x_{jis}^m - \sum_{i\in\mathcal{K}} \sum_{m\in\mathcal{M}} x_{ijs}^m = 0, \qquad \forall j \in \mathcal{L} \tag{16}$$

$$\sum_{j:ij\in\mathcal{A}} \sum_{m\in\mathcal{M}} x_{ijs}^m - \eta_i y_i \leq 0, \qquad \forall i \in \mathcal{P} \tag{17}$$

$$w x_{ijs}^m - W_m t_{ijs}^m \leq 0, \qquad \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{18}$$

$$x_s \in \mathbb{R}_+^{|\mathcal{A}|\times|\mathcal{M}|},\ t_s \in \mathbb{R}_+^{|\mathcal{A}|\times|\mathcal{M}|},\ z_s, e_s \in \mathbb{R}_+^{|\mathcal{K}|}. \tag{19}$$

285 Note that equations (12) - (19) are scenario-specific versions of (2) - (9) and that relatively complete recourse is provided by the slack variables in (13) and (14). To incorporate the third stage and consider the carbon tax uncertainty, we introduce the RC and AARC formulations of the recourse problem in the following section.

290 *3.2. Robust Counterparts of the Recourse Problems*

The robust counterpart of the recourse problem is to find an optimal solution that satisfies all constraints for any carbon tax rate $\tilde{\alpha} \in \mathcal{U}$. We define the RC of (11) - (19) as:

$$Q_{RC}(y,s) = \min_{u_s,x_s,t_s,z_s,e_s} u_s \quad \text{such that } \forall \tilde{\alpha} \in \mathcal{U}, \tag{20}$$

$$\sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} h^m t^m_{ijs} + \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} g^m \beta_{ij} x^m_{ijs} + \sum_{k\in\mathcal{K}}(\theta z_{ks} + \zeta e_{ks})$$

$$+ w\tilde{\alpha} \sum_{ij\in\mathcal{A}} \beta_{ij} \sum_{m\in\mathcal{M}} \tau^m x^m_{ijs} \leq u_s, \tag{21}$$

$$\sum_{ij\in\mathcal{A}} \left(h^m t^m_{ijs} + g^m \beta_{ij} x^m_{ijs} + w\tilde{\alpha}\beta_{ij}\tau^m x^m_{ijs} \right) \geq L^m, \ \forall m \in \mathcal{M} \tag{22}$$

$$(13) \ - \ (19), \tag{23}$$

where $u_s \in \mathbb{R}, x_s, t_s, z_s$ and e_s are all here-and-now decisions regarding the carbon tax uncertainty. However, the RC formulation may provide an overly conservative solution by requiring all decision variables to be feasible for all values of $\tilde{\alpha}$ in the uncertainty set.

To obtain a less conservative solution, we assume that x_s and t_s are adjustable variables; i.e., their values can be determined after the tax rate uncertainty is resolved (Ben-Tal et al., 2004). We assume the uncertain value $\tilde{\alpha}$ falls in a box uncertainty set. Specifically, $\tilde{\alpha} = \bar{\alpha} + \xi\hat{\alpha}$, where the perturbation scalar ξ varies in the set $\Xi_p \equiv \{\xi | \ |\xi| \leq \rho\}$. Without loss of generality, the adjustable variables can be adjusted to the perturbation scalar ξ instead of $\tilde{\alpha}$ (Ben-Tal et al., 2004). Therefore, the ARC is written as follows:

$$Q_{ARC}(y,s) = \min_{u_s,z_s,e_s} u_s \ \text{ such that } \forall \xi \in \Xi_p, \exists x_s(\xi) \text{ and } t_s(\xi) \text{ such that} \tag{24}$$

$$(21) - (23), \tag{25}$$

where variables x_s and t_s are functions of the uncertain parameter ξ. Generally, ARC models cannot be solved efficiently even in fixed recourse cases. A tractable approximation is provided by the AARC, where the adjustable variables are restricted to be affine functions of the uncertain parameters (Ben-Tal et al., 2004). Here, we set $x^m_{ijs} = v^m_{ij(0)s} + \xi v^m_{ij(1)s}$ and $t^m_{ijs} = \pi^m_{ij(0)s} + \xi\pi^m_{ij(1)s}$, where $v_{(0)s}, v_{(1)s}, \pi_{(0)s}$ and $\pi_{(1)s}$ are non-adjustable variables.

Under this restriction, the product flows and transportation capacity decisions in the ARC (24) - (25) are replaced by an AARC$_{x,t}$ given by:

$$Q_{AARC_{x,t}}(y,s) = \min_{u_s,v_s,\pi_s,z_s,e_s} u_s \quad \text{such that } \forall \xi \in \Xi_p, \tag{26}$$

$$\sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} h^m\left(\pi^m_{ij(0)s} + \xi\pi^m_{ij(1)s}\right) + \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} g^m\beta_{ij}\left(v^m_{ij(0)s} + \xi v^m_{ij(1)s}\right)$$
$$+\sum_{k\in\mathcal{K}}(\theta z_{ks} + \zeta e_{ks}) + w(\bar{\alpha} + \xi\hat{\alpha})\sum_{ij\in\mathcal{A}}\beta_{ij}\sum_{m\in\mathcal{M}}\tau^m\left(v^m_{ij(0)s} + \xi v^m_{ij(1)s}\right) \le u_s, \tag{27}$$

$$\sum_{ij\in\mathcal{A}}\Bigg(h^m\left(\pi^m_{ij(0)s} + \xi\pi^m_{ij(1)s}\right) + g^m\beta_{ij}\left(v^m_{ij(0)s} + \xi v^m_{ij(1)s}\right)$$
$$+ w(\bar{\alpha} + \xi\hat{\alpha})\beta_{ij}\tau^m\left(v^m_{ij(0)s} + \xi v^m_{ij(1)s}\right)\Bigg) \ge L^m, \ \forall m \in \mathcal{M} \tag{28}$$

$$\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}}\left(v^m_{jk(0)s} + \xi v^m_{jk(1)s}\right) + z_{ks} = d^n_{ks}, \qquad \forall k \in \mathcal{K} \tag{29}$$

$$\sum_{i\in\mathcal{L}}\sum_{m\in\mathcal{M}}\left(v^m_{ki(0)s} + \xi v^m_{ki(1)s}\right) + e_{ks} = d^o_{ks}, \qquad \forall k \in \mathcal{K} \tag{30}$$

$$\sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}}\left(v^m_{ji(0)s} + \xi v^m_{ji(1)s}\right) - \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\left(v^m_{ij(0)s} + \xi v^m_{ij(1)s}\right) = 0, \qquad \forall j \in \mathcal{J} \tag{31}$$

$$\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\left(v^m_{ji(0)s} + \xi v^m_{ji(1)s}\right) - \sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}}\left(v^m_{ij(0)s} + \xi v^m_{ij(1)s}\right) = 0, \qquad \forall j \in \mathcal{L} \tag{32}$$

$$\sum_{j:ij\in\mathcal{A}}\sum_{m\in\mathcal{M}}\left(v^m_{ij(0)s} + \xi v^m_{ij(1)s}\right) - \eta_i y_i \le 0, \qquad \forall i \in \mathcal{P} \tag{33}$$

$$w\left(v^m_{ij(0)s} + \xi v^m_{ij(1)s}\right) - W_m\left(\pi^m_{ij(0)s} + \xi\pi^m_{ij(1)s}\right) \le 0, \qquad \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{34}$$

$$\left(v^m_{ij(0)s} + \xi v^m_{ij(1)s}\right) \ge 0, \qquad \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{35}$$

$$v_s \in \mathbb{R}^{|\mathcal{A}|\times|\mathcal{M}|}, \ \pi_s \in \mathbb{R}^{|\mathcal{A}|\times|\mathcal{M}|}, \ z_s, e_s \in \mathbb{R}^{|\mathcal{K}|}_+, \ u_s \in \mathbb{R}. \tag{36}$$

The purpose of the AARC formulation is to produce less conservative solutions than the RC. However, because uncertainty affects only (21) when $L^m = 0$ for all $m \in \mathcal{M}$ in constraint (22) and is thus constraint-wise, RC (20)-(23) satisfies Theorem 2.1 of Ben-Tal et al. (2004), which defines conditions under which the objectives of RC and ARC are equal.

The AARC$_{x,t}$ model (26)-(36) has uncertain recourse because it allows product flows to adjust to the carbon tax rate. For solution it can be converted into a semi-definite program (SDP) as detailed in the Appendix. To investigate the value of allowing product flows to be adjustable, we also formualte a robust product-flow version of the model, where only the transportation capacities are adjustable, as follows:

$$Q_{AARC_t}(y,s) = \min_{u_s,x_s,\pi_s,z_s,e_s} u_s \quad \text{such that } \forall \xi \in \Xi_p, \tag{37}$$

$$\begin{aligned}&\sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} h^m \left(\pi^m_{ij(0)s} + \xi\pi^m_{ij(1)s}\right) + \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} g^m \beta_{ij} x^m_{ijs} \\ &+ \sum_{k\in\mathcal{K}}(\theta z_{ks} + \zeta e_{ks}) + w(\bar{\alpha}+\xi\hat{\alpha})\sum_{ij\in\mathcal{A}}\beta_{ij}\sum_{m\in\mathcal{M}}\tau^m x^m_{ijs} \le u_s,\end{aligned} \tag{38}$$

$$\sum_{ij\in\mathcal{A}}\left(h^m\left(\pi^m_{ij(0)s}+\xi\pi^m_{ij(1)s}\right) + g^m\beta_{ij}x^m_{ijs} + w(\bar{\alpha}+\xi\hat{\alpha})\beta_{ij}\tau^m x^m_{ijs}\right) \ge L^m, \ \forall m \in \mathcal{M} \tag{39}$$

$$(13)-(17), \text{ and} \tag{40}$$

$$w x^m_{ijs} - W_m\left(\pi^m_{ij(0)s} + \xi\pi^m_{ij(1)s}\right) \le 0, \qquad \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{41}$$

$$x_s \in \mathbb{R}_+^{|\mathcal{A}|\times|\mathcal{M}|},\ \pi_s \in \mathbb{R}^{|\mathcal{A}|\times|\mathcal{M}|},\ z_s, e_s \in \mathbb{R}_+^{|\mathcal{K}|},\ u_s \in \mathbb{R}. \tag{42}$$

Here, all the decisions are second-stage decision variables once t_s has been replaced by its affine function of ξ.

Assuming $\tilde{\alpha}$ belongs to a box uncertainty set, the RC (20)-(23) with $L^m > 0$ for some $m \in \mathcal{M}$ in (22) and the AARC$_t$ model satisfy the conditions of Haddad-Sisakht & Ryan (2016), which are loosely described as: the model contains at least two binding constraints at optimality of the RC formulation and an

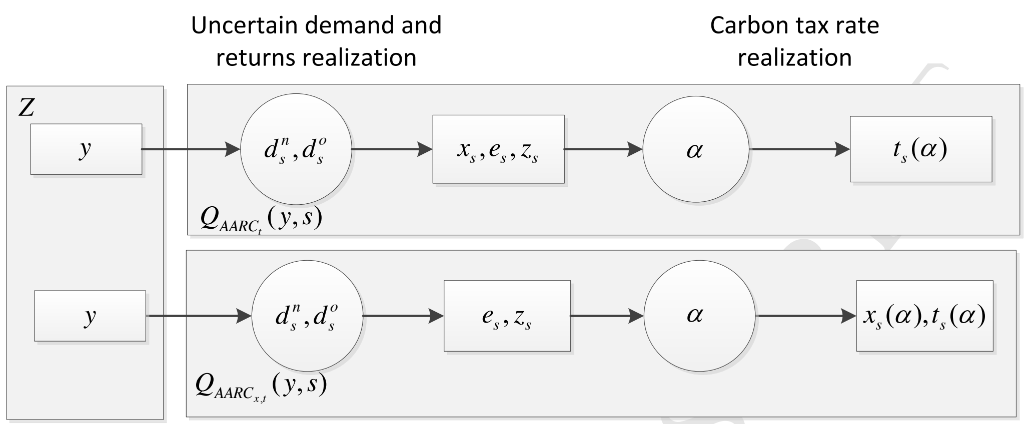


Figure 2: Three stages of hybrid robust/stochastic model where small boxes show decision variables and outer boxes define the portions of model to find Z and $Q(y, s)$ values. The upper and lower figure show AARC$_t$ and AARC$_{x,t}$ models, respectively.

adjustable variable in both constraints with implicit bounds from above and below for different extreme values in the uncertainty set. Therefore, the affinely adjustable models (26)-(36) and (37)-(42) could result in a less conservative solution than (20)-(23), depending on the parameter values.

3.3. Integration of robust optimization and stochastic programming

Figure 2 illustrates the three stages of our hybrid robust/stochastic model for both AARC$_t$ and AARC$_{x,t}$ with probabilistic scenarios for demands and returns and an uncertainty set for the carbon tax rate.

These models are summarized as follows:

$$Z_{RC} = \min_{y \in \{0,1\}^{|\mathcal{P}|}} \sum_{i \in \mathcal{P}} c_i y_i + \sum_{s \in \mathcal{S}} P_s Q_{RC}(y, s) \tag{43}$$

where the affine adjustable versions are:

$$Z_{AARC_{x,t}} = \min_{y \in \{0,1\}^{|\mathcal{P}|}} \sum_{i \in \mathcal{P}} c_i y_i + \sum_{s \in \mathcal{S}} P_s Q_{AARC_{x,t}}(y,s) \tag{44}$$

$$Z_{AARC_t} = \min_{y \in \{0,1\}^{|\mathcal{P}|}} \sum_{i \in \mathcal{P}} c_i y_i + \sum_{s \in \mathcal{S}} P_s Q_{AARC_t}(y,s) \tag{45}$$

Problem (44) is currently intractable because of the binary variables y but its subproblems $Q_{AARC_{x,t}}(y,s)$ can be solved as semi-definite programs (see the Appendix). The numerical studies in Section 4.3 show little difference between $Q_{AARC_{x,t}}(y,s)$ and $Q_{AARC_t}(y,s)$. Therefore, in the sequel we focus attention on models (43) and (45), which are mixed-integer linear programs.

Problems (43) and (45) can be solved directly but, with large numbers of scenarios and potential facilities, this approach would become computationally cumbersome. We use a multi-cut version of Benders decomposition (BD) to decompose the problem into master and sub-problems (Birge & Louveaux, 2011). Because the recourse problems are always feasible since these models have relatively complete recourse, only optimality cuts are generated. The master problem is:

$$Z_{AARC_t} = \min_{y \in \{0,1\}^{|\mathcal{P}|}, \delta_s} \sum_{i \in \mathcal{P}} c_i y_i + \sum_{s \in \mathcal{S}} \delta_s \tag{46}$$

s.t. Optimality cuts,

where $\delta_s \in \mathbb{R}$ is a lower bound on the objective value for sub-problem s.

The decision variables in the master problem are the binary facility investment variables y and lower bounds on the subproblem objectives. The subproblems for each scenario $s \in \mathcal{S}$ with optimal objective value Σ_s (where $\Sigma_s = Q_{RC}(y,s)$ or $\Sigma_s = Q_{AARC_t}(y,s)$ for the RC or AARC$_t$ formulation, respectively) minimize upper bounds on transportation, shortage and emission costs for a given y. The BD algorithm solves the master problem and subproblems iteratively. If $\Sigma_s > \delta_s$ in master problem (46), an optimality cut is

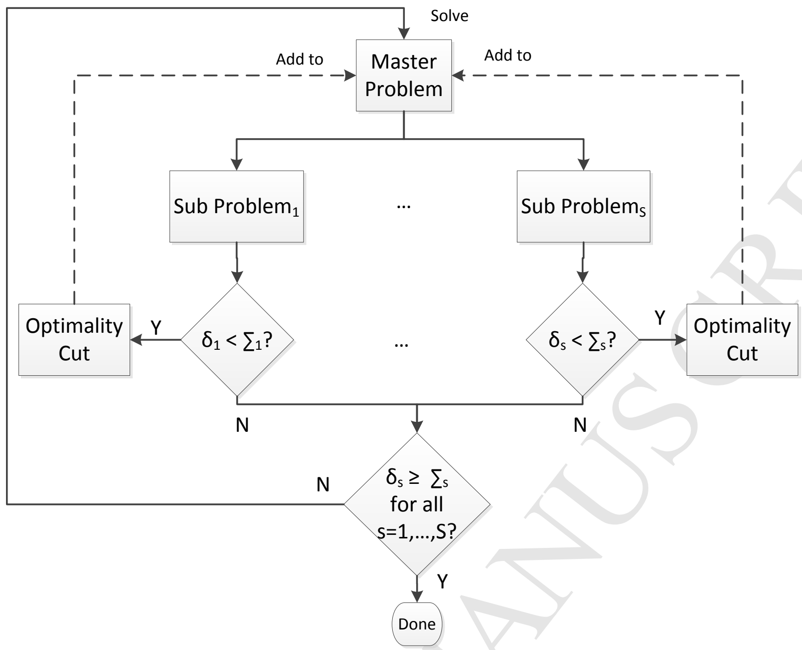


Figure 3: Iterations of the multi-cut Benders decomposition algorithm

added. The algorithm continues until $\Sigma_s \leq \delta_s$ for all scenarios $s \in \mathcal{S}$ (Birge & Louveaux, 2011). Figure 3 illustrates the iterations of the BD algorithm.

Master problem (46) can be solved with usual MILP solvers. An optimality cut for a scenario is obtained using the dual solution of the corresponding subproblem. Each subproblem is an AARC$_t$ or RC formulation with carbon tax polyhedral uncertainty set that can converted to an explicit linear program (LP) by defining additional constraints and variables as explained in Ben-Tal et al. (2004). Their duals can be obtained using the approach of Beck & Ben-Tal (2009). By denoting the dual variables of constraints (38), (39), (13) - (17), and (41), respectively, as λ_1 to λ_8, the dual of subproblem (37) - (42) is as follows:

$$\Sigma_s^D = \max_{\lambda} \sum_{i \in \mathcal{P}} \eta_i y_i \lambda_{7i} + \sum_{k \in \mathcal{K}} (d_{ks}^m \lambda_{3k} + d_{ks}^o \lambda_{4k}) + \sum_{m \in \mathcal{M}} L^m \lambda_{2m} \tag{47}$$

$$\text{s.t.} \quad -\lambda_1 = P_s, \tag{48}$$

$$h^m(\lambda_1 + \lambda_{2m}) - W_m \lambda_{8ij}^m = 0, \qquad \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{49}$$

$$h^m \xi(\lambda_1 + \lambda_{2m}) - W_m \xi \lambda_{8ij}^m = 0, \qquad \text{for some } \xi \in \Xi_p, \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{50}$$

$$\theta\lambda_1 + \lambda_{3k} \leq 0, \qquad \forall k \in \mathcal{K} \tag{51}$$

$$\zeta\lambda_1 + \lambda_{4k} \leq 0, \qquad \forall k \in \mathcal{K} \tag{52}$$

$$(g^m \beta_{ij} + \tilde{\alpha} w \beta_{ij} \tau^m)(\lambda_1 + \lambda_{2m}) + w\lambda_{8ij}^m + \lambda_{5j} + \lambda_{7i} \leq 0, \qquad \text{for some } \tilde{\alpha} \in \mathcal{U}, \forall ij \in (\mathcal{I}, \mathcal{J}), m \in \mathcal{M} \tag{53}$$

$$(g^m \beta_{jk} + \tilde{\alpha} w \beta_{jk} \tau^m)(\lambda_1 + \lambda_{2m}) + w\lambda_{8jk}^m + \lambda_{3k} - \lambda_{5j} \leq 0, \qquad \text{for some } \tilde{\alpha} \in \mathcal{U}, \forall jk \in (\mathcal{J}, \mathcal{K}), m \in \mathcal{M} \tag{54}$$

$$(g^m \beta_{kl} + \tilde{\alpha} w \beta_{kl} \tau^m)(\lambda_1 + \lambda_{2m}) + w\lambda_{8kl}^m + \lambda_{4k} + \lambda_{6l} + \lambda_{7l} \leq 0, \qquad \text{for some } \tilde{\alpha} \in \mathcal{U}, \forall kl \in (\mathcal{K}, \mathcal{L}), m \in \mathcal{M} \tag{55}$$

$$(g^m \beta_{li} + \tilde{\alpha} w \beta_{li} \tau^m)(\lambda_1 + \lambda_{2m}) + w\lambda_{8li}^m - \lambda_{6l} + \lambda_{7i} \leq 0, \qquad \text{for some } \tilde{\alpha} \in \mathcal{U}, \forall li \in (\mathcal{L}, \mathcal{I}), m \in \mathcal{M} \tag{56}$$

$$\lambda_1 \in \mathbb{R}_-, \lambda_2 \in \mathbb{R}_+^{|\mathcal{M}|}, \lambda_3, \lambda_4 \in \mathbb{R}^{|\mathcal{K}|}, \lambda_5 \in \mathbb{R}^{|\mathcal{J}|}, \lambda_6 \in \mathbb{R}^{|\mathcal{L}|}, \lambda_7 \in \mathbb{R}_-^{|\mathcal{P}|}, \lambda_8 \in \mathbb{R}_-^{|\mathcal{A}| \times |\mathcal{M}|} \tag{57}$$

370 If $\Sigma_s > \delta_s$, the following optimality cut is added to the master problem for the next iteration:

$$\sum_{i\in\mathcal{P}} \eta_i y_i \lambda_{7i}^* + \sum_{k\in\mathcal{K}} (d_{ks}^n \lambda_{3k}^* + d_{ks}^o \lambda_{4k}^*) + \sum_{m\in\mathcal{M}} L^m \lambda_{2m}^* \leq \delta_s, \tag{58}$$

where the left-hand-side is Σ_s^D from (47).

4. Computational Experiments

To explore the effects of adjustability and uncertainty on the decisions and their costs, we present a computational experiment based on randomly generated instances with realistic parameter values as described in Section 4.1. In Section 4.2, we assess the value of adjustability. Using the full hybrid stochastic/robust model for the second and third stages, we compare the optimal expected worst case costs when both transportation capacities and product flows can adjust to the carbon tax rate to their counterparts when only the transportation capacities are adjustable. Observing little difference, we focus the rest of the study on the hybrid model with adjustability only in transportation capacities. We evaluate RC and AARC$_t$ solutions assuming deterministic demands and returns to explore the effects of adjustability in the transportation mode capacity on design decisions and the role of the transportation cost lower bounds. By comparing the results of the RC and AARC$_t$ models with various sizes of the uncertainty set we observe the impact of tax rate uncertainty on the transportation capacities and facility locations. In Section 4.3 we evaluate the combined effect of uncertainties in hybrid robust/stochastic instances where demand and return quantities as well as the carbon tax rate are integrated. The effects of uncertainties on the optimal cost, facility investment, and transportation mode choices are also evaluated and the superiority of the hybrid formulation over the deterministic one is shown.

4.1. Parameter definitions

We randomly generate the locations of potential facilities and customers within a 3500 km × 2000 km rectangle, and use Euclidean distance. In most of the computational experiments, we assume there are five potential facilities for each of plants, warehouses, and collection centers. The goal is to satisfy 20 customers in different locations. We consider larger numbers of potential facilities for some of instaces. The uniform distributions of data generators for the fixed costs c_i and capacities η_i of potential factories, warehouses and collection centers are shown in Table 1.

Table 1: The generator distributions for fixed cost and capacities of potential facilities

	Fixed Cost c_i(\$1000)	Capacities η_i (units of product)
Factories	Uniform$[1000, 4000]$	Uniform$[3000, 6000]$
Warehouses	Uniform$[500, 1500]$	Uniform$[3000, 7000]$
Collection Centers	Uniform$[500, 1500]$	Uniform$[600, 900]$

Based on research studies such as Levinson et al. (2004) and Mallidis et al. (2012), many approaches have been used to estimate truck operating costs which depend on fuel, repair and maintenance, tire, depreciation, and labor cost. Levinson et al. (2004) conducted a survey to identify the average cost per kilometer for the average truckload, which they found to be \$0.69/km. In addition, several sources such as Coyle et al. (2011) and a white paper by Armstrong Associates, Inc. (2009) approximate that 70 to 90 percent of truck operating costs are variable and 10 to 30 percent are fixed costs. More specifically, the latter stated that variable costs include those parameters changing within a year, such as direct labor, fuel, insurance, rented equipment, and maintenance. Fixed costs, which include depreciation, building leased/purchased, management/salespeople, and overhead, are usually steady over a year.

In our computational experiments, only road transport modes are considered. Modes 1, 2 and 3, respectively, represent light, mid-size and heavy trucks, with the relevant parameter values shown in Table 2. Estimated weights W_m of light, mid-size, and heavy trucks are derived from U.S. government documents (U.S. Department of Transportation, 2000). The estimated unit transportation costs of the modes g^m (per km per ton) for the trucks are calculated based on Byrne et al. (2006). We assume each unit is a pallet with 1.1 ton weight. The fixed operating cost h^m per unit of capacity for each road mode is calculated based on approximately 20% of total truck operating costs (Coyle et al., 2011). We calculate the total cost of each truck by multiplying the average distance between facilities by the maximum weight of each truck divided by 0.80. Therefore, the fixed costs for different instances depend on the randomly generated distances. The h^m values for the deterministic instances of Section 4.1 are provided in the fourth column of Table 2. The carbon emission factor, τ^m, of road transport mode m depends on the mode as well as its vehicle condition, maintenance, roads, type of fuel, and many other factors. The values that we used, shown in the last column of Table 2, are based on data from The Network for Transport and Environment (2014). Heavy trucks usually have lower emission rate per ton but more capacity than light trucks.

Table 2: The estimated parameters of transportation modes

Mode, m	W_m (tons)	g^m($/units of product-km)	h^m($/unit of transportation)	τ^m (tons/km-ton)
1	8.9	0.0213	68	0.00025
2	15.2	0.0211	115	0.00018
3	19.6	0.0240	169	0.00012

We generate three scenarios for demands: low, medium and high, where for each customer, k, the low demand d^n_{k1} is generated according to a normal distribution with mean value 400 units and standard deviation 100; i.e., N(400,100). We assume the medium and high demands of customer k are $d^n_{k2} = d^n_{k1} + 100$ and $d^n_{k3} = d^n_{k2} + 100$. Independent of demands, returned products d^o_k are obtained by multiplying a rate of return Rt_k generated from N(0.2 , 0.1) by demands; i.e., $d^o_{ks} = Rt_k d^n_{ks}$. Shortage costs θ and ζ for unmet demands and uncollected returned products usually exceed other components such as production and transportation costs (Absi & Kedad-Sidhoum, 2008). Therefore, after calculating the maximum amount of fixed and variable cost of transporting one unit to a customer, shortage cost are randomly generated according to Uniform[1000, 1500], where the lower bound is larger than the maximum cost of transporting a single unit.

For the nominal value of the uncertain carbon tax rate $\bar{\alpha}$, the carbon tax rate of British Columbia in 2012 (Sumner et al., 2009) is used. All MILPs are solved by CPLEX on a computer with 8 GB RAM and Intel Core i7 2.00 GHz CPU.

4.2. Impact of adjustability

We first explore the value of allowing product flows, x, as well as transportation capacities, t, adjust to the value of the carbon tax rate. We solve the AARC$_t$ formulation (45) and save the optimal facility investment decisions, y, as well as the optimal expected worst-case cost $\sum_{s\in\mathcal{S}} P_s Q_{AARC_t}(y,s)$. Then, assuming those investment decisions have been implemented, for each scenario we solve the AARC$_{x,t}$ formulation (26)-(36) of the second and third stages as a semi-definite program (see the Appendix). We compute $\sum_{s\in\mathcal{S}} P_s Q_{AARC_{x,t}}(y,s)$ for comparison. Finally we compute the gaps between these expected worst case costs and the corresponding expected worst-case RC recourse cost from (20) - (23).

Figure 4 shows that, for a large range of widths of tax uncertainty set $\hat{\alpha}$, the difference between the gaps are negligible; i.e., adjustability of the product flows in addition to the transportation capacities has very little impact given a fixed set of facilities. Figure 5 shows the same comparison for a fixed level of uncertainty as the lower bound on the cost of transportation mode 1 increases. While the expected worst-case recourse cost differences between AARC$_{x,t}$ and AARC$_t$ are negligible, the average computational times of the SDP model for the second and third stages are about 450 times those of the three-stage MILP model in this instances. Therefore, we focus attention on the more computationally efficient AARC$_t$ formulation for the remainder of the experiments.

We next evaluate and compare the RC and AARC$_t$ solutions for different sizes of the uncertainty set and values of lower bounds on transportation costs, assuming deterministic demands and product returns. The carbon tax uncertainty set is $\tilde{\alpha} = \bar{\alpha} + \xi\hat{\alpha}$ where the nominal value $\bar{\alpha} = 30$ and the deviation value $\hat{\alpha}$ ranges from 0 to 30 with $|\xi| \leq 1$. The deterministic model of carbon

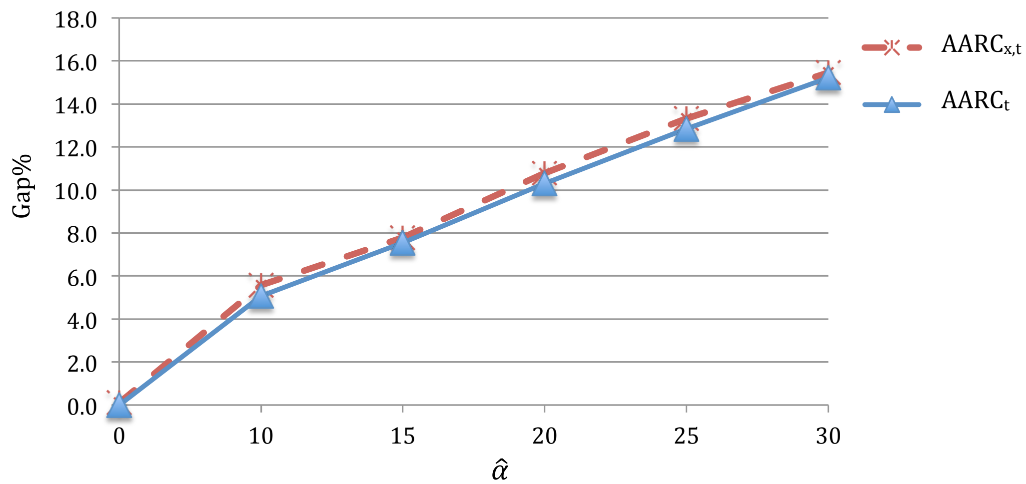


Figure 4: The comparison of the percentage gaps, computed as $(\sum_{s\in\mathcal{S}} P_s Q_{RC}(s) - \sum_{s\in\mathcal{S}} P_s Q_{AARC}(y,s))/\sum_{s\in\mathcal{S}} P_s Q_{RC}(y,s)$ between $\text{AARC}_{x,t}$ and AARC_t for various values of the carbon uncertainty set radius $\hat{\alpha}$ when $\bar{\alpha} = 30, L^2 = L^3 = 0$, and $L^1 = 0.75M$.

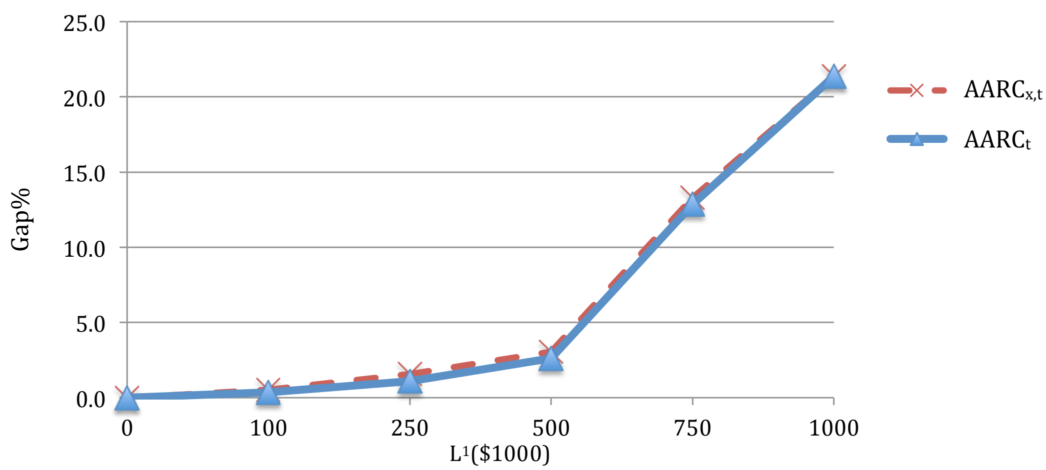


Figure 5: The comparison of the percentage gaps, computed as $(\sum_{s\in\mathcal{S}} P_s Q_{RC}(s) - \sum_{s\in\mathcal{S}} P_s Q_{AARC}(y,s))/\sum_{s\in\mathcal{S}} P_s Q_{RC}(y,s)\%$ between $\text{AARC}_{x,t}$ and AARC_t for various values of L^1 when uncertainty set $\hat{\alpha} = 25$, $\bar{\alpha} = 30$, and $L^2 = L^3 = 0$.

Table 3: The comparison between RC and AARC$_t$ when $\bar{\alpha} = 30$, and $L^1 = L^2 = L^3 = 0$ for different values of $\hat{\alpha}$. The % use of mode m is $\sum_{ij \in \mathcal{A}} x_{ij}^m / \left(\sum_{\mu \in \mathcal{M}} \sum_{ij \in \mathcal{A}} x_{ij}^\mu \right)$ %.

	% use of mode				
$\hat{\alpha}$	m=1	m=2	m=3	Z_{AARC_t}	$(Z_{RC} - Z_{AARC_t})/Z_{RC}\%$
0	0	100	0	11643265	0
10	0	100	0	11700130	0
15	0	100	0	11728563	0
20	0	100	0	11756995	0
25	0	77	23	11782611	0
30	0	70	30	11806341	0

tax uncertainty has $\hat{\alpha} = 0$, and deterministic demands and returns are assumed by considering a single scenario that represents the expected value of demand and return quantities for each customer.

The RC (43) and AARC$_t$ (45) solutions for different values of $\hat{\alpha}$ with $\bar{\alpha} = 30$ and $L^1 = L^2 = L^3 = 0$ are compared in Table 3. In this table, the total use of three modes by summing over total product flows of all arcs are shown to be the same for both RC and AARC$_t$ formulations. As shown in the last column, there is no difference between the RC and the AARC$_t$ solutions because uncertainty is constraint-wise. Mode 2 is used in most cases when there is no lower bound on transportation cost but, as the uncertainty of carbon tax increases, the use of lower-emitting transportation mode 3 increases.

Table 4 shows the results of setting the lower bound, L^1, on transportation and emission costs of mode 1 to \$1M with $L^2 = L^3 = 0$. The RC and the AARC$_t$ solutions for different tax rate uncertainty sets are compared for $\bar{\alpha} = 30$. The facility configuration is the same for both RC and AARC$_t$. The difference between the RC and AARC$_t$ objective values increases with the uncertainty of the carbon tax rate. In all of these instances, the use of mode 2 or 3, with lower emission cost, is higher in the AARC$_t$ solution than in the RC solution.

Tables 5 and 6 illustrate the differences between RC and AARC$_t$ solutions and optimal objective values when the lower bound on transportation cost of modes 1 and 3, respectively, vary from \$0.1M to \$1M. The AARC$_t$ solution is progressively less conservative than the RC solution as the lower bound on

Table 4: The comparison between RC and AARC$_t$ when $\bar{\alpha} = 30, L^2 = L^3 = 0$, and $L^1 = 1M$ for different values of $\hat{\alpha}$. The % use of mode m is $\sum_{ij \in \mathcal{A}} x_{ij}^m / \left(\sum_{\mu \in \mathcal{M}} \sum_{ij \in \mathcal{A}} x_{ij}^\mu \right)$%.

		% use of mode			$(Z_{RC} - Z_{AARC_t})/Z_{RC}$%
$\hat{\alpha}$	Types	m=1	m=2	m=3	
0	RC	97	3	0	0.00
	AARC$_t$	97	3	0	
10	RC	100	0	0	0.24
	AARC$_t$	94	6	0	
15	RC	100	0	0	0.62
	AARC$_t$	92	8	0	
20	RC	100	0	0	0.99
	AARC$_t$	91	9	0	
25	RC	96	0	4	1.23
	AARC$_t$	90	0	10	
30	RC	99	0	1	1.48
	AARC$_t$	88	0	12	

the cost of either transportation mode increases. However, the RC and AARC$_t$ objective differences with the mode 1 lower bound (Table 5) are higher than with the mode 3 lower bound (Table 6) because mode 1 has the higher emission rate.

4.3. Combined effect of uncertainties on decision variables

In this section we compare the solutions to the hybrid robust/stochastic formulations (43) and (45) with different levels of uncertainty of both types.

Effect of demand/return quantity uncertainty on the optimal cost

Solutions to the deterministic (single-scenario) and stochastic models are compared as follows. To implement the deterministic model, the expected values of the scenarios for the demand and return quantities are used. Let $\bar{d}$ be the expected value of the demand and return vector. The optimal value of the deterministic problem can be expressed as EV= Z_{AARC_t} from (45) with deterministic $\bar{d}$. The EV solution for the facility configuration is denoted by $\bar{y}(\bar{d})$. For the recourse problem (RP), the optimal value is denoted as RP= Z_{AARC_t} obtained using the three scenarios. When the performance of

Table 5: The comparison between RC and AARC$_t$ when $\bar{\alpha} = 30, \hat{\alpha} = 10, L^2 = L^3 = 0$ for different values of L^1.

L^1(\$1000)	Types	% use of mode m=1	m=2	m=3	$(Z_{RC} - Z_{AARC_t})/Z_{RC}\%$
100	RC	20	80	0	0.01
	AARC$_t$	19	81	0	
250	RC	40	60	0	0.02
	AARC$_t$	36	64	0	
500	RC	69	31	0	0.05
	AARC$_t$	62	38	0	
750	RC	88	12	0	0.09
	AARC$_t$	82	18	0	
1000	RC	100	0	0	0.24
	AARC$_t$	94	06	0	

Table 6: The comparison between RC and AARC$_t$ when $\bar{\alpha} = 30, \hat{\alpha} = 10, L^1 = L^2 = 0$ for different values of L^3.

L^3(\$1000)	Types	% use of mode m=1	m=2	m=3	$(Z_{RC} - Z_{AARC_t})/Z_{RC}\%$
100	RC	0	95	05	0.00
	AARC$_t$	0	95	05	
250	RC	0	87	13	0.00
	AARC$_t$	0	88	12	
500	RC	0	68	32	0.01
	AARC$_t$	0	72	28	
750	RC	0	40	60	0.02
	AARC$_t$	0	46	54	
1000	RC	0	3	97	0.04
	AARC$_t$	0	17	83	

Table 7: Evaluating hybrid robust/stochastic AARC$_t$ solution with robust AARC$_t$ solution when $\bar{\alpha} = 30$, and $L^2 = L^3 = 0$, for different values of $\hat{\alpha}$.

	$L^1 = 0$			$L^1 = 1M$		
$\hat{\alpha}$	Stochastic (RP)	EEV	$\frac{VSS}{RP}\%$	Stochastic (RP)	EEV	$\frac{VSS}{RP}\%$
0	12,391,806	12,432,293	0.33	12,463,191	12,555,984	0.74
10	12,448,955	12,483,124	0.27	12,534,298	12,601,637	0.53
15	12,475,871	12,508,539	0.26	12,567,575	12,623,956	0.45
20	12,502,786	12,533,954	0.25	12,600,414	12,645,983	0.36
25	12,527,455	12,557,362	0.24	12,631,885	12,666,767	0.28
30	12,548,517	12,578,992	0.24	12,662,188	12,687,443	0.20

the deterministic solution $\bar{y}(\bar{d})$ is evaluated in the stochastic model, we obtain $EEV = \sum_{i \in \mathcal{P}} c_i \bar{y}_i(\bar{d}) + \sum_{s \in \mathcal{S}} P_s Q_{AARC_t}(\bar{y}(\bar{d}), s)$.

The amount of savings that results from solving the stochastic model, called the value of the stochastic solution (VSS), equals EEV$-$RP (Birge & Louveaux, 2011). The costs of RP and EEV and their comparisons for the AARC$_t$ model are shown in Tables 7 and 8. For example, the VSS with the nominal value of the carbon tax rate $\hat{\alpha} = 0$ and $L^1 = 0$ in Table 7, is EEV $-$ RP $= 40,487$ which is 0.33% of RP.

The results in Table 7 indicate that the savings from solving the stochastic program compared to the deterministic model decrease as the carbon tax rate uncertainty increases. Table 8 shows the cost savings from the stochastic model's solution for different values of lower bounds on modes 1 and 3. The highest cost savings are observed for the highest values of each lower bound.

Effect of uncertainty on facility investment and transportation mode choice

Figure 6 shows the facility configuration of the solution of AARC$_t$ (45) when demands and returns are deterministic and $\hat{\alpha} = 10$ assuming five potential facilities of each type and 20 customers. In addition, the lower bounds on transportation and emission costs for all three modes are assumed to be zero. In this instance, three plants, three warehouses, and two collection centers are opened. Figure 7 shows the facility configuration of the same instance as in Figure 6 but with stochastic demands and returns. In the latter solution the

Table 8: Evaluating hybrid robust/stochastic AARC_t solution with robust AARC_t solution when $\bar{\alpha} = 30, \hat{\alpha} = 10$, and $L^2 = 0$ for different values of L^1 and L^3.

$L(\$1000)$		Stochastic (RP)	EEV	$\frac{VSS}{RP}\%$
$(L^3 = 0), L^1:$	100	12,455,944	12,489,521	0.27
	250	12,467,874	12,501,327	0.27
	500	12,489,066	12,521,598	0.26
	750	12,511,519	12,543,165	0.25
	1000	12,534,298	12,601,637	0.53
$(L^1 = 0), L^3:$	100	12,451,178	12,485,342	0.27
	250	12,454,847	12,488,945	0.27
	500	12,461,284	12,496,485	0.28
	750	12,469,413	12,506,054	0.29
	1000	12,486,465	12,568,429	0.65

Table 9: The comparison among "mean ± standard error" of the AARC_t solutions of ten randomly generated instances of parameters with different values of $\bar{\alpha}$ when $L^1 = \$1.5M, L^2 = L^3 = 0$ and $\hat{\alpha} = 10$.

	Average use of modes(%)			Average opened facilities		
$\bar{\alpha}$	m=1	m=2	m=3	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{J}\rvert$	$\lvert\mathcal{K}\rvert$
20	91 ± 1.2	9 ± 1.2	0 ± 0.0	8.1 ± 0.2	7.6 ± 0.2	4.4 ± 0.7
35	87 ± 1.9	13 ± 1.9	0 ± 0.0	7.8 ± 0.2	7.5 ± 0.2	4.1 ± 0.7
50	85 ± 0.9	6 ± 1.3	9 ± 1.4	8.1 ± 0.3	7.6 ± 0.2	3.7 ± 0.6

numbers of both warehouses and collection centers are decreased from three to two facilities, and one plant has moved to a different location compared to the solution of the deterministic model in Figure 6.

Table 9 displays the solutions of larger instances as the nominal carbon tax $\bar{\alpha}$ increases from 20 to 50. For each carbon tax uncertainty level, we randomly generated ten instances of demands, returns, fixed costs, and capacities from their distributions, maintaining a fixed number, 20, of potential facilities of each type to satisfy 70 customers. The results in Table 9 show that by increasing the nominal value of the carbon tax rate, the use of modes with lower emission rate would significantly increase. However, unlike the results found in Gao & Ryan (2014), the number of opened facilities does not significantly change.

Table 10 shows the results for 20 trials of the same experiment to compare the solutions for stochastic and deterministic demands and returns of the AARC_t formulation. We randomly generated the probabilities of scenarios 1 and 2 from

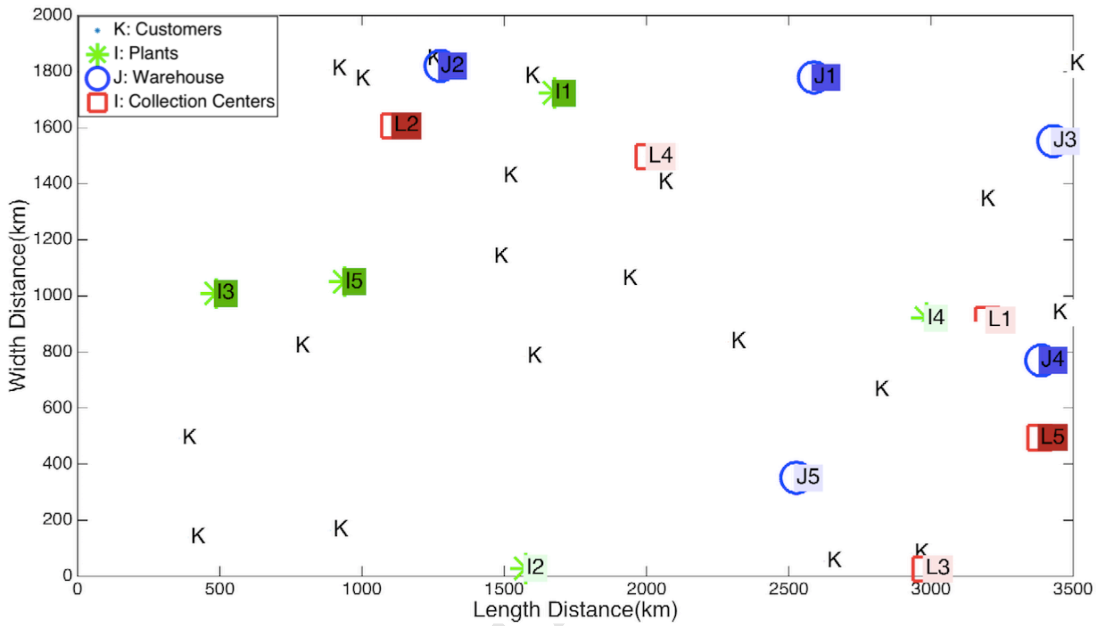


Figure 6: Facility configuration of RC or AARC$_t$ solution when demands and returns are deterministic and $\hat{\alpha} = 10$ and $L^1 = L^2 = L^3 = 0$. Opened facilities are shown in darker color.

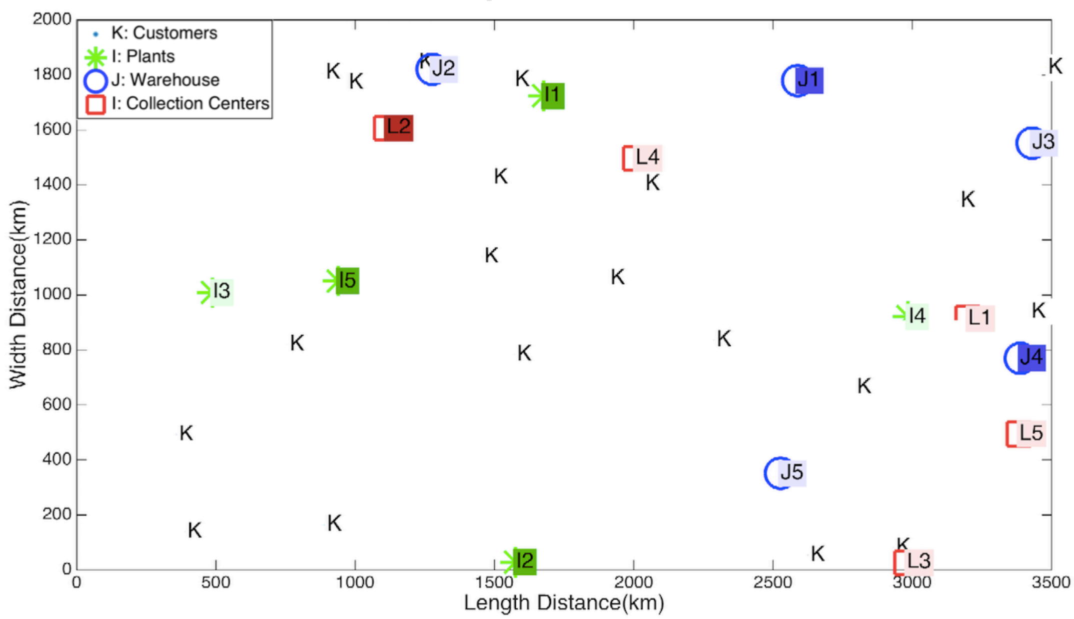


Figure 7: Facility configuration of RC or AARC$_t$ solution when demands and returns are uncertain and $\hat{\alpha} = 10$ and $L^1 = L^2 = L^3 = 0$. Opened facilities are shown in darker color.

Table 10: The comparison among "mean ± standard error" of the AARC$_t$ solutions of 20 randomly generated instances of parameters between deterministic and stochastic demands and returns when $L^1 = \$1.5M, L^2 = L^3 = 0, \bar{\alpha} = 50$ and $\hat{\alpha} = 30$.

	Average use of modes(%)			Average opened facilities		
	m=1	m=2	m=3	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{J}\rvert$	$\lvert\mathcal{K}\rvert$
Stochastic	96 ± 0.6	0 ± 0.0	4 ± 0.6	8.35 ± 0.2	7.95 ± 0.1	3.4 ± 0.6
Deterministic	91 ± 1.0	0 ± 0.0	9 ± 1.0	9.25 ± 0.2	8.75 ± 0.1	3.4 ± 0.5

Uniform$[0.3, 0.35]$ and set $P_3 = 1-(P_1+P_2)$. The results show that the solution to the stochastic version opens fewer facilities compared to the solution to the deterministic model but the use of modes with lower capacity or higher emission rate increases.

To see how the number of opened facilities is affected by adjustability assuming 20 potential facilities of each type to satisfy 70 customers, Figure 8 shows the total number of opened facilities for four different randomly generated instances. We assumed higher demands to represent longer periods by setting the mean and standard deviation of demands to be 100 and 10000 units, respectively, and the demands in the medium and high scenarios to be 10000 and 20000 units, respectively, more than those in the low scenario. Also the facility capacities for plants and warehouses were randomly generated from Unif$[1M, 2M]$, and for the collection centers from Unif$[0.1M, 0.2M]$. The results in Figure 8 indicate that by increasing the nominal value of the carbon tax rate, the number of opened facilities is increased. However, there are values of $\bar{\alpha}$ for which the solution of AARC$_t$ would open fewer facilities compared to the RC solution. Thus, the AARC$_t$ model provides a less conservative solution not only in terms of transportation modes but also in terms of facility investment while satisfying all demands and returns.

5. Conclusions

In this paper, we formulated a hybrid robust/stochastic model for CLSC network design that is subject to uncertainty in demands and returned products. We used probabilistic scenarios for the quantities of demands and returned

ACCEPTED MANUSCRIPT

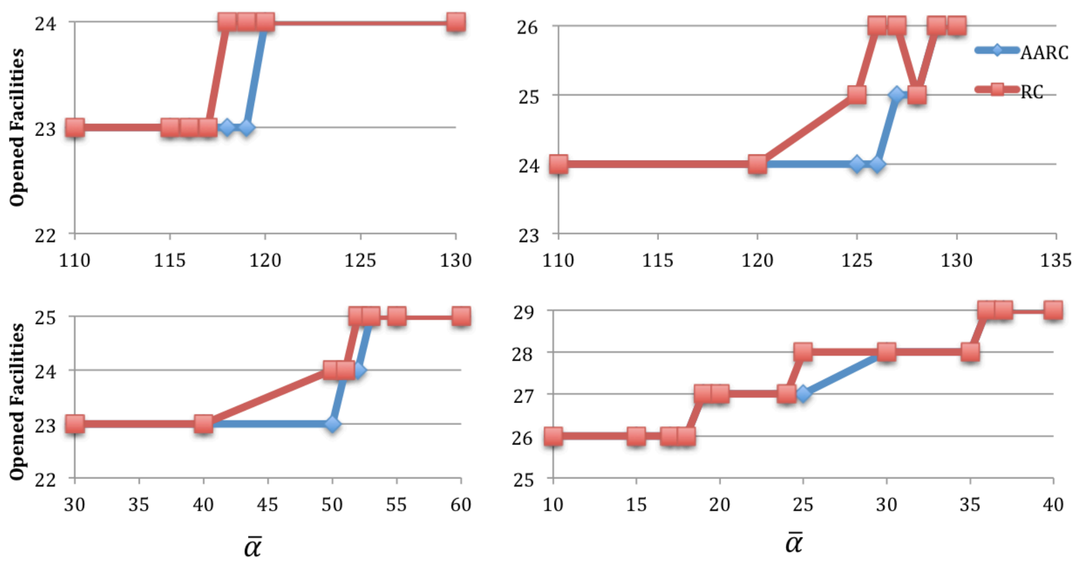


Figure 8: Total number of opened facilities of RC and AARC_t solution when $\bar{\alpha}$ is increasing in horizontal axes and $\hat{\alpha} = 10$, $L^1 = \$100M, L^2 = L^3 = 0$.

products where the first stage decisions are facility configuration and product flows are determined in the second stage after demand and return quantities are realized. The model structure accommodates carbon tax policy by ensuring that the resulting solutions of facility configuration and product flows are robust to the uncertain carbon tax rate. The transportation capacities as the third stage decisions are assumed to be affine functions of the carbon tax rate for tractable yet less conservative solution to the problem.

In computational experiments, we illustrated the reduced conservatism provided by affine adjustability in the robust counterpart. We analyzed the solutions of the RC, AARC_t and $\text{AARC}_{x,t}$ formulations with different levels of uncertainty in the carbon tax rate and lower bounds on the transportation and emission costs of different modes. The results confirm the intuitive understanding that the total expected cost in the worst case of the carbon tax rate is decreased by increasing the utilization of transportation modes with higher capacity per unit and lower emission rate. This behavior is consistent across different levels of the lower bounds on transportation and emission costs by mode.

Imposing a lower bound on the mode with highest emission rate maximizes the cost difference between the RC and AARC_t solutions. The numerical results of the comparison between the SDP model with both product flows and transportation capacities adjustable to the carbon tax rate and the LP model with only transportation capacities adjustable indicates that the benefit of the more complex model is negligible. The number of opened facilities in AARC_t solutions is decreased under uncertainty in demands and returns, which indicates the potential for over-investment in facilities if this source of uncertainty is ignored. When there is uncertainty in demands and returns, the numbers of opened facilities do not vary with the nominal value of carbon tax, but the optimal use of modes with lower emission rates increases. In addition, the AARC_t solution opens fewer facilities and more highly utilizes modes with lower emission rates than the RC solution. That is, adjustability in the transportation capacity by mode can substitute for facility investment as a hedge against carbon tax rate uncertainty.

Suggestions for future research include expanding the formulation to multiple time periods that would accommodate temporal variability in demands, returns and carbon tax rates, with multiple stages of decision-making. In addition, explicitly modeling inventories in the facilities to the problem could be a useful extension to examine the tradeoff between emission and inventory costs.

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant No. 1130900.

Appendix

In this section, we illustrate how to model the $\text{AARC}_{x,t}$ formulation (44) with adjustability in product flows as well as transportation capacities using semi-definite programming (SDP). To evaluate $Q_{AARC_{x,t}}(y, s)$ (26)-(36) for a single scenario and given value of y, we use the SDP approach explained in Ben-Tal et al. (2004). The functions of adjustable variables x and t are considered to be $x^m_{ijs} = v^m_{ij(0)s} + \xi v^m_{ij(1)s}$ and $t^m_{ijs} = \pi^m_{ij(0)s} + \xi \pi^m_{ij(1)s}$. In the uncertain parameter $\tilde{\alpha} = \bar{\alpha} + \xi\hat{\alpha}$, the perturbation scalar ξ belongs to a box uncertainty set, which is a special case of the ellipsoidal uncertainty set, $\xi \in \chi \equiv \left\{\xi | \ \xi^T \Delta \xi \leq \rho^2\right\}$ with $\Delta = 1$ and $\rho = 1$.

The SDP reformulation to find $Q_{AARC_{x,t}}(y,s)$ is as follows:

$$SDP\_Q_{AARC_{x,t}}(y,s) = \min_{u_s,v_s,\pi_s,z_s,e_s} u_s \tag{59}$$

$$\text{s.t.} \quad \begin{pmatrix} -\Gamma_{1s} + \rho^{-2}\gamma_{1s}\Delta & -V_{1s}/2 \\ -V_{1s}/2 & -\sigma_{1s} - \gamma_{1s} \end{pmatrix} \succcurlyeq 0, \tag{60}$$

$$\begin{pmatrix} -\Gamma_{2ms} + \rho^{-2}\gamma_{2ms}\Delta & -V_{2ms}/2 \\ -V_{2ms}/2 & -\sigma_{2ms} - \gamma_{2ms} \end{pmatrix} \succcurlyeq 0, \forall m \in \mathcal{M} \tag{61}$$

$$\begin{pmatrix} \rho^{-2}\gamma_{3ks}\Delta & \sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} v^m_{jk(1)s}/2 \\ \sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} v^m_{jk(1)s}/2 & \begin{array}{c}\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} v^m_{jk(0)s}+ \\ z_{ks} - d^n_{ks} - \gamma_{3ks}\end{array} \end{pmatrix} = 0, \forall k \in \mathcal{K} \tag{62}$$

$$\begin{pmatrix} \rho^{-2}\gamma_{4ks}\Delta & \sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} v^m_{jk(1)s}/2 \\ \sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} v^m_{jk(1)s}/2 & \begin{array}{c}\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} v^m_{jk(0)s}+ \\ e_{ks} - d^0_{ks} - \gamma_{4ks}\end{array} \end{pmatrix} = 0, \forall k \in \mathcal{K} \tag{63}$$

$$\begin{pmatrix} \rho^{-2}\gamma_{5js}\Delta & \begin{array}{c}\big(\sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}} v^m_{ji(1)s}- \\ \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} v^m_{ij(1)s}\big)/2\end{array} \\ \begin{array}{c}\big(\sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}} v^m_{ji(1)s}- \\ \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} v^m_{ij(1)s}\big)/2\end{array} & \begin{array}{c}\sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}} v^m_{ji(0)s}- \\ \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} v^m_{ij(0)s} - \gamma_{5js}\end{array} \end{pmatrix} = 0, \forall j \in \mathcal{J} \tag{64}$$

$$\begin{pmatrix} \rho^{-2}\gamma_{6js}\Delta & \begin{array}{c}\big(\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} v^m_{ji(1)s}- \\ \sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}} v^m_{ij(1)s}\big)/2\end{array} \\ \begin{array}{c}\big(\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} v^m_{ji(1)s}- \\ \sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}} v^m_{ij(1)s}\big)/2\end{array} & \begin{array}{c}\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} v^m_{ji(0)s}- \\ \sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}} v^m_{ij(0)s} - \gamma_{6js}\end{array} \end{pmatrix} = 0, \forall j \in \mathcal{L} \tag{65}$$

$$\begin{pmatrix} \rho^{-2}\gamma_{7ijms}\Delta & (W_m\pi^m_{ij(1)s} - wv^m_{ij(1)s})/2 \\ (W_m\pi^m_{ij(1)s} - wv^m_{ij(1)s})/2 & W_m\pi^m_{ij(0)s} - wv^m_{ij(0)s} - \gamma_{7ijms} \end{pmatrix} \succcurlyeq 0, \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{66}$$

$$\begin{pmatrix} \rho^{-2}\gamma_{8is}\Delta & \sum_{j\in\mathcal{P}\setminus\mathcal{N}}\sum_{m\in\mathcal{M}} v^m_{ij(1)s}/2 \\ \sum_{j\in\mathcal{P}\setminus\mathcal{N}}\sum_{m\in\mathcal{M}} v^m_{ij(1)s}/2 & \eta_i y_i - \sum_{j\in\mathcal{P}\setminus\mathcal{N}}\sum_{m\in\mathcal{M}} v^m_{ij(0)s} - \gamma_{8is} \end{pmatrix} \succcurlyeq 0, \quad \forall i \in \mathcal{N}, \mathcal{N} \subset \mathcal{P} \tag{67}$$

$$\begin{pmatrix} \rho^{-2}\gamma_{9ijms}\Delta & v^m_{ij(1)s}/2 \\ v^m_{ij(1)s}/2 & v^m_{ij(0)s} - \gamma_{9ijms} \end{pmatrix} \succcurlyeq 0, \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{68}$$

$$v_s \in \mathbb{R}^{|\mathcal{A}|\times|\mathcal{M}|},\ \pi_s \in \mathbb{R}^{|\mathcal{A}|\times|\mathcal{M}|},\ z_s, e_s \in \mathbb{R}_+^{|\mathcal{K}|},\ u_s \in \mathbb{R}. \tag{69}$$

where $\gamma >= 0$ in all the constraints and auxiliary variables Γ, V and σ in (60) and (61) are as follows:

$$\begin{aligned}
\Gamma_{1s} =& \hat{\alpha}^T w \sum_{ij\in\mathcal{A}} \beta_{ij} \sum_{m\in\mathcal{M}} \tau^m v^m_{ij(1)s}, \\
V_{1s} =& \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} h^m \pi^m_{ij(1)s} + \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} g^m \beta_{ij} v^m_{ij(1)s} + \\
& w \sum_{ij\in\mathcal{A}} \beta_{ij} \sum_{m\in\mathcal{M}} \tau^m \left(\hat{\alpha} v^m_{ij(0)s} + \bar{\alpha} v^m_{ij(1)s}\right), \\
\sigma_{1s} =& \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} h^m \pi^m_{ij(0)s} + \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} g^m \beta_{ij} v^m_{ij(0)s} + \sum_{k\in\mathcal{K}} (\theta z_{ks} + \zeta e_{ks}) + \\
& \bar{\alpha} w \sum_{ij\in\mathcal{A}} \beta_{ij} \sum_{m\in\mathcal{M}} \tau^m v^m_{ij(0)s} - u_s,
\end{aligned} \tag{70}$$

$$\begin{aligned}
\Gamma_{2ms} =& \hat{\alpha}^T w \sum_{ij\in\mathcal{A}} \beta_{ij} \tau^m v^m_{ij(1)s},\ \forall m \in \mathcal{M} \\
V_{2ms} =& \sum_{ij\in\mathcal{A}} h^m \pi^m_{ij(1)s} + \sum_{ij\in\mathcal{A}} g^m \beta_{ij} v^m_{ij(1)s} + w \sum_{ij\in\mathcal{A}} \beta_{ij} \tau^m \left(\hat{\alpha} v^m_{ij(0)s} + \bar{\alpha} v^m_{ij(1)s}\right),\ \forall m \in \mathcal{M}, \\
\sigma_{2ms} =& \sum_{ij\in\mathcal{A}} h^m \pi^m_{ij(0)s} + \sum_{ij\in\mathcal{A}} g^m \beta_{ij} v^m_{ij(0)s} + \sum_{k\in\mathcal{K}} (\theta z_{ks} + \zeta e_{ks}) + \\
& \bar{\alpha} w \sum_{ij\in\mathcal{A}} \beta_{ij} \tau^m v^m_{ij(0)s} - L^m, \qquad \forall m \in \mathcal{M},
\end{aligned} \tag{71}$$

The experiments in the paper are implemented with SDPT3 solver using

YALMIP package in Matlab platform to solve the SDP model.

References

Absi, N., & Kedad-Sidhoum, S. (2008). The multi-item capacitated lot-sizing problem with setup times and shortage costs. *European Journal of Operational Research*, *185*, 1351–1374. doi:10.1016/j.ejor.2006.01.053.

Allevi, E., Gnudi, A., Konnov, I., & Oggioni, G. (2016). Evaluating the effect of environmental regulations on a closed-loop supply chain network: a variational inequality approach. *Optimization online*, . Available at: http://www.optimization-online.org/DB_FILE/2016/06/5475.pdf.

Armstrong Associates, Inc. (2009). Carrier procurement insights, trucking company volume, cost and pricing tradeoffs. *Armatrong and Associates, Inc.*, .

Beck, A., & Ben-Tal, A. (2009). Duality in robust optimization: Primal worst equals dual best. *Operations Research Letters*, *37*, 1–6. doi:10.1016/j.orl.2008.09.010.

Ben-Tal, A., Goryashkoy, A., Guslitzerz, E., & Nemirovski, A. (2004). Adjustable robust solutions of uncertain linear programs. *Mathematical Programming*, *99*.

Ben-Tal, A., & Nemirovski, A. (1998). Robust convex optimization. *Mathematics of Operations Research*, *23*.

Ben-Tal, A., & Nemirovski, A. (1999). Robust solutions of uncertain linear programs. *Operations Research Letters*, *25*.

Ben-Tal, A., & Nemirovski, A. (2000). Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical Programming*, *88*, 411–424.

Benders, J. F. (1962). Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, *4*, 238 – 252.

Benjaafar, S., Li, Y., & Daskin, M. (2013). Carbon footprint and the management of supply chains: Insights from simple models. *IEEE Transactions on Automation Science and Engineering*, *10*, 99–116. doi:110.1109/TASE.2012.2203304.

Behringer, C., Rutherford, T. F., & Tol, R. S. J. (2009). The EU 20/20/2020 targets: An overview of the EMF22 assessment. *Energy Economics*, *31*, S268–S273. doi:10.1016/j.eneco.2009.10.010.

Birge, J. R., & Louveaux, F. (2011). *Introduction to Stochastic Programming*. Springer Series in Operations Research and Financial Engineering (2nd ed.). London: Springer.

Bloemhof-Ruwaard, J., Van der Laan, E., & Beijer, C. (2011). Sustainable inland transportation. *International Journal of Business Insights and Transformation*, *3*, 26 – 33. URL: http://library.wur.nl/WebQuery/wurpubs/410574. doi:urn:nbn:nl:ui:32-410574.

Byrne, J., Waegel, A., Haney, B., Tobin, D., Alleng, G., Karki, J., & Suarez, J. (2006). Pathways to a US hydrogen economy: Vehicle fleet diffusion scenarios. *Technical report, Newark, DE: Center for Energy and Environmental Policy*, .

Chaabane, A., Ramudhin, A., & Paquet, M. (2012). Design of sustainable supply chains under the emission trading scheme. *International Journal of Production Economics*, *135*, 37–49. doi:10.1016/j.ijpe.2010.10.025.

Coyle, J. J., Novak, R. A., Gibson, B., & Bardi, E. J. (2011). *Transportation: A Supply Chain Perspective*. (7th ed.). New York: Cengage Learning.

El Ghaoui, L., El Oustry, F., & Lebret, H. (1998). Robust solutions to uncertain semidefinite programs. *SIAM Journal on Optimization*, *9*.

El Ghaoui, L., & Lebret, H. (1997). Robust solutions to least-squares problems with uncertain data. *SIAM Journal on Matrix Analysis and Applications*, *18*.

Fahimnia, B., Sarkis, J., Dehghanian, F., Banihashemi, N., & Shams, R. (2013). The impact of carbon pricing on a closed-loop supply chain: an Australian case study. *Journal of Cleaner Production*, *59*, 210–225. doi:10.1016/j.jclepro.2013.06.056.

Fareeduddin, M., Hassan, A., Syed, M. N., & Selim, S. Z. (2015). The impact of carbon policies on closed-loop supply chain network design. *Procedia CIRP*, *26*, 335–340. doi:10.1016/j.procir.2014.07.042.

Fu, M., & Kelly, J. A. (2012). Carbon related taxation policies for road transport: Efficacy of ownership and usage taxes, and the role of public transport and motorist cost perception on policy outcomes. *Transport Policy*, *22*, 57–69. doi:10.1016/j.tranpol.2012.05.004.

Gao, N., & Ryan, S. M. (2014). Robust design of a closed-loop supply chain network for uncertain carbon regulations and random product flows. *EURO Journal on Transportation and Logistics*, *3*, 1–30. doi:10.1007/s13676-014-0043-7.

Guide, D. R., & Van Wassenhowe, L. N. (2002). The reverse supply chain. Smart manufacturers are designing efficient processes for reusing their products. *Harvard Business Review*, *80*, 25–26.

Haddad-Sisakht, A., & Ryan, S. M. (2016). Conditions under which adjustability lowers the cost of a robust linear program. Available at: http://lib.dr.iastate.edu/imse_reports/3/.

Hickman, O., Robin Ashiru, & Banister, D. (2011). Transitions to low carbon transport futures: strategic conversations from london and delhi. *Journal of Transport Geography*, *19*, 1553–1562. doi:10.1016/j.jtrangeo.2011.03.013.

Hoen, K., Tan, T., Fransoo, J., & van Houtum, G. (2010). Effect of carbon emission regulations on transport mode selection in supply chains. *Flexible Services and Manufacturing Journal*, *26*, 170–195. doi:10.1007/s10696-012-9151-6.

Keyvanshokooh, E., Ryan, S. M., & Kabir, E. (2016). Hybrid robust and stochastic optimization for closed-loop supply chain network design using accelerated benders decomposition. *European Journal of Operational Research*, *249*, 76–92.

Levinson, D., Corbett, M., & Hashami., M. (2004). Operating costs for trucks. *http://nexus.umn.edu/papers/truckoperatingcosts.pdf*, .

Mallidis, I., Dekker, R., & Vlachos, D. (2012). The impact of greening on supply chain design and cost: a case for a developing region. *Journal of Transport Geography*, *22*, 118–128. doi:10.1016/j.jtrangeo.2011.12.007.

Metcalf, G. E. (2009). Market-based policy options to control U.S. greenhouse gas emissions. *Journal of Economic Perspectives*, *23*, 5–27.

Paksoy, T., Turan Bektas, & Ozceylan, E. (2011). Operational and environmental performance measures in a multi-product closed-loop supply chain. *Transportation Research Part E: Logistics and Transportation Review*, *47*, 532–546. doi:10.1016/j.tre.2010.12.001.

Pan, S., Ballot, E., & Fontane, F. (2010). The reduction of greenhouse gas emissions from freight transport by pooling supply chains. *International Journal of Production Economics*, *143*. doi:10.1016/j.ijpe.2010.10.023.

Soyster, A. L. (1972). Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations Research*, *21*.

Sumner, J., Bird, L., & Smith, H. (2009). *Carbon Taxes: A Review of Experience and Policy Design Considerations*. Technical Report National Renewable Energy Laboratory. Available at: http://www.nrel.gov/docs/fy10osti/47312.pdf.

Tao, Z. G., Guang, Z. Y., Hao, S., Song, H. J., & Xin, D. G. (2015). Multi-period closed-loop supply chain network equilibrium with carbon emission constraints. *Resources, Conservation and Recycling*, *104*, 354–365. doi:10.1016/j.resconrec.2015.07.016.

The Network for Transport and Environment (2014). URL: http://www.ntmcalc.se/index.html.

Turban, E., King, D., Lee, J. K., Liang, T.-P., & Turban, D. C. (2015). Order fulfillment along the supply chain. In E. Turban, D. King, J. K. Lee, T.-P. Liang, & D. C. Turban (Eds.), *Electronic Commerce: A Managerial and Social Networks Perspective*. Springer Texts in Business and Economics.

U.S. Department of Transportation (2000). The U.S. Department of Transportation's comprehensive truck size and weight study. *U.S. Department of Transportation*, *2*.

U.S. Environmental Protection Agency (2016a). Inventory of U.S. greenhouse gas emissions and sinks: 1990-2014. URL: https://www.epa.gov/sites/production/files/2016-04/documents/us-ghg-inventory-2016-main-text.pdf.

U.S. Environmental Protection Agency (2016b). Sources of greenhouse gas emissions, U.S. Environmental Protection Agency. URL: http://www.epa.gov/climatechange/ghgemissions/sources/transportation.html viewed (Sept 2016).

U.S. Environmental Protection Agency (2017). The social cost of carbon. Estimating the benefits of reducing greenhouse gas emissions. URL: https://www.epa.gov/climatechange/social-cost-carbon viewed (Jan 2017).

Vahdani, B., & Mohammadi, M. (2015). A bi-objective interval-stochastic robust optimization model for designing closed loop supply chain network with multi-priority queuing system. *International Journal of Production Economics*, *170*, 67–78.

Vahdani, B., Tavakkoli-Moghaddam, R., Modarres, M., & Baboli, A. (2012). Reliable design of a forward/reverse logistics network under uncertainty: A robust-M/M/c queuing model. *Transportation Research Part E: Logistics and Transportation Review*, *48*, 1152–1168. doi:10.1016/j.tre.2012.06.002.

Xinhuanet (2013). China to introduce carbon tax: official. http://news.xinhuanet.com/english/china/2013-02/19/c_132178898.htm.

Xu, Z., Pokharel, S., Elomri, A., & Mutlu, F. (2017). Emission policies and their analysis for the design of hybrid and dedicated closed-loop supply chains. *Journal of Cleaner Production*, *142*, 4152–4168. doi:10.1016/j.jclepro.2016.09.192.

Yuzhong, M., & Guangming, Y. (2012). A study of outsourcing and self-run business decision- making in the distribution transportation of international logistics. *Energy Procedia*, *16*, 965–970. doi:10.1016/j.egypro.2012.01.154.

Zakeri, A., Dehghanian, F., Fahimni, B., & Sarkis, J. (2015). Carbon pricing versus emissions trading: A supply chain planning perspective. *International Journal of Production Economics*, *164*, 197–205.

Zeballos, L. J., Gomes, M. I., Barbosa-Povoa, A. P., & Novais, A. Q. (2012). Addressing the uncertain quality and quantity of returns in closed-loop supply chains. *Computers and Chemical Engineering*, *47*, 237–247. doi:10.1016/j.compchemeng.2012.06.034.